MA453
Homework b

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1. $D_4 = \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$

   so, ord$(2) = \{r^2, s, sr, sr^2, sr^3\}$ and, ord$(4) = \{r^2, r^3\}$

   so, the subgroups of size 2:

   $\{e, r^2\}, \{e, s\}, \{e, sr\}, \{e, sr^2\}, \{e, sr^3\}$

   so, the subgroups of size 4:

   must be C_4 or KV_4

   $C_4 = \{e, r, r^2, r^3\}, \{e, r^2, sr, sr^3\}, \{e, r^2, s, sr^2\}$

2. Every reflection of D_n can be written as as sr^k for some k

   so, $(sr)^i(sr)^j = s(r^i s)r^j = (sr^i s)r^j = r^{-i} \cdot r^j = r^{j-i}$

   which is a reflection.

3. a) $e \in H, xex^{-1} = e \in H'$

b) Suppose that $y, z \in H'$

then $y = xhx^{-1}$ and $z = xgx^{-1}$ for some $h, g \in H$

$y \cdot z = (xhx^{-1})(xgx^{-1}) = xh(x^{-1}x)gx^{-1} = xhgx^{-1}$

so $hg \in H \Rightarrow yz \in H'$

c) $y^{-1} = (xhx^{-1})^{-1} = x^{-1}h^{-1}x = xh^{-1}x^{-1}$

so, $h^{-1} \in H \Rightarrow y^{-1} \in H'$

d) define $\phi : H \to H'$ by $\phi(h) = xhx^{-1}$

$\phi(hg) = xhgx^{-1} = (xhx^{-1})(xgx^{-1}) = \phi(h)\phi(g)$

so relabeling matches.

e) Suppose that $x = e$

then, $eHe^{-1} = \{ehe^{-1} : h \in H\} = \{h : h \in H\}$

f) Suppose G is abelian then,

$xhx^{-1} = hxx^{-1} = h$ because $x, h \in G$

so, $xHx^{-1} = \{xhx^{-1} : h \in H\} = \{h : h \in H\} = H$

true for all $x \in G$, so only itself to conjugate.

4. Opposite faces, rotations: 90, 180, 270; $3 \cdot 3 = 9$.

   Opposite vertices, rotations: 120, 240; $4 \cdot 2 = 8$

   Opposite edges, rotation: 180; $1 \cdot 6 = 6$

   So, rigid motions 24

   So, one forms a coset of size 24 and the other coset must contain exactly as many

   So, there should be 48 elements in the group of symmetries of the cube.