MA 453
Homework b

1. so, $c^3 = (1, 4, 7)(2, 5, 8)(3, 6, 9)$ just a shift by 3

   so we want $\pi(1) = 1, \pi(2) = 4, \pi(3) = 7, \pi(4) = 2, \pi(5) = 5, \pi(6) = 8, \pi(7) = 3, \pi(8) = 6, \pi(9) = 9$

   so we have $\sigma = (1, 4, 7, 2, 5, 8, 3, 6, 9)$ and we we shift by 3 we get

   $\sigma^3 = (1, 2, 3)(4, 5, 6)(7, 8, 9)$

2. Try S_3, so $|S_3| = 3! = 6$, but we need an element with order 6 but the max n-cycle in S_3 is 3, which means it can't be cyclic. Ouch!

   Same for S_n, where $n \geq 3$

   Try S_2, so $|S_2| = 2! = 2$

   $S_2 = \{e, (1, 2)\}$

   $e \to \{e\}$

   $(1, 2) \to \{(1, 2), (1, 2)^2\} = \{(1, 2), e\}$

   so, S_2 is cyclic and by the hint so is S_1

   which are the only symmetric groups that are cyclic.

3. Begin with

$$c\pi^{-1} = c(\pi^{-1}(j)) = \begin{cases} j, & \text{if } j \notin \{a_1, ..., a_s\} \\ a_{k+1}, & \text{if } j \in \{a_1, ..., a_s\} \text{ with } j = a_k \text{ for some } 1 \leq k \leq s \end{cases}$$

Similarly,

$$\pi c \pi^{-1}(j) = \begin{cases} j, & \text{if } j \notin \{a_1, ..., a_s\} \\ a_k, & \text{if } j \in \{a_1, ..., a_s\} \text{ with } j = a_k \text{ for some } 1 \leq k \leq s \end{cases}$$

so, we get $(\pi_{a_1}, ..., \pi_{a_s}) \underbrace{(j_1)...(j_n)}_{\text{all the } j \notin \{a_1, ..., a_s\}}$

which is just, $(\pi_{a_1}, ..., \pi_{a_s})$

4. $\sigma(1,2)(3,4)\sigma^{-1} = (\sigma(1), \sigma(2))(\sigma(3), \sigma(4)) = (1,3)(2,4)$

so, $\sigma(1) = 1, \sigma(2) = 3, \sigma(3) = 2, \sigma(4) = 4$

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 2 & 4 \end{pmatrix}$$