MA 453
Homework b

1. $\phi(n) = 42$

   We know that $\phi(n) = n - 1$ if n is prime.

   so, $n - 1 = 42 \Rightarrow n = 43$ and 43 is prime!

   so, one $n = 43$

   Next, there is no n such that $\phi(n) = 15$

   By definition for $n > 2$, $\phi(n)$ is even, so can't get an n such that $\phi(n) = 15$. Ouch!

2. Essentially, we want $\phi(n) = 18$

   We know that $\phi(n) = n - 1$ if n is prime.

   so, $n - 1 = 18 \Rightarrow n = 19$ and 19 is prime!

   so, $p^{a-1}(p-1) = 18$

   try $p = 3$: $3^{a-1}(3-1) = 18 \iff 3^a = 27 \iff a = 3$

   so, $n = 27$

   try $p = 2$: $2^{a-1}(2-1) = 18 \iff 2^a = 36$, Ouch! We can't!

   Next, we know that $\phi(19) = 18$ and $\phi(2) = 1$ and $gcd(2, 19) = 1$

   so, $\phi(19 \cdot 2) = \phi(19) \cdot \phi(2) = 18 \Rightarrow n = 38$

   Similarly, we know that $\phi(27) = 18$ and $\phi(2) = 1$ and $gcd(2, 27) = 1$

   so, $\phi(27 \cdot 2) = \phi(27) \cdot \phi(2) = 18 \Rightarrow n = 54$

3. Suppose that ord$(a) = t$

   So, ord$(a^k) = \dfrac{t}{gcd(t,k)}$

   $\dfrac{t}{gcd(t,k)} = 18 \iff t = 18 \cdot gcd(t,k)$

   $t = 18 \cdot \underbrace{gcd(t,k)}_{\text{either 1 or 5}} \Rightarrow t = 18, 90$

   so, possible orders for a are 18, 90

   Similarly, $t = 20 \cdot gcd(t,5)$

   $t = 20, 100$

   thus, possible orders for a are $20, 100$

4. (a) $x = 21s + 3$ for some $s \in \mathbb{Z}$

$3 + 21s \equiv 4(\text{mod } 10)$

$21s \equiv 1(\text{mod } 10)$

$s \equiv 1(\text{mod } 10)$ because $21(\text{ mod } 10) \equiv 1$

$s - 1 = 10t \Rightarrow s = 10t + 1$

$x = 21(10t + 1) + 3 = 210t + 24$

so, $x \equiv 24 \text{ mod } 210$

(b) $y = 10s + 9$ for some $s \in \mathbb{Z}$

$9 + 10s \equiv 2(\text{mod } 3)$

$10s \equiv -7(\text{mod } 10) \equiv 2(\text{mod } 3)$

$s \equiv 2(\text{mod } 3)$ because $10(\text{ mod } 3) \equiv 1$

$s - 2 = 3t \Rightarrow s = 3t + 2$

$y = 10(3t + 2) + 9 = 29 + 30t$

so, $y \equiv 29 \text{ mod } 30$

Next, $29 + 30t \equiv 5(\text{ mod })7$

$30t \equiv -24(\text{ mod } 7) \iff 2t \equiv 4(\text{mod} 7) \iff t \equiv 2(\text{ mod } 7)$

so, $t = 2 + 7u$

$29 + 30(2 + 7u) \equiv 5(\text{ mod } 7)$

$89 + 210u \equiv 5(\text{ mod } 7) \Rightarrow y \equiv 89(\text{ mod } 210)$