MA 453
Week 0b

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1. Yitang Zhang

2. a) $a, b, c, d = 1, 6, 8, 9$

b) $a, b, c, d = 10, 9, 12, 1$

3. Suppose that $a, k \in \mathbb{Z}$

   So, $2ak \in \mathbb{Z}$

   Similarly, $k^2 - a^2 \in \mathbb{Z}$

   So, $(2ak)^2 + (k^2 - a^2)^2 = 4a^2k^2 - k^2a^2 + k^4 - a^2k^2 + a^4 = k^4 + 2k^2a^2 + a^4 = (k^2 + a^2)^2$

   So, $k^2 + a^2 \in \mathbb{Z}$

   and by definition, $\{2ak, k^2 - a^2, k^2 + a^2\}$ is a Pythagorean triple.

4. Suppose there are some nonnegative integer sequences $\{a_n\}, \{k_n\}$ so that, $x_n = 2a_nk_n, y_n = a_n^2 - k_n^2, z_n = a_n^2 + k_n^2$

   So, (x_n, y_n, z_n) is always a Pythagorean triple by exercise 3.

   Let P_n be a point s.t $P_n = (\frac{x_n}{z_n}, \frac{y_n}{z_n})$

   which is in the unit circle because $(\frac{x_n}{z_n})^2 + (\frac{y_n}{z_n})^2 = 1$

   So, $P_n = (\frac{x_n}{z_n}, \frac{y_n}{z_n}) = (\frac{2a_nk_n}{a_n^2+k_n^2}, \frac{a_n^2-k_n^2}{a_n^2+k_n^2})$

   and let P be a point in the unit circle s.t $P = (x, y) = (\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2})$ for some $t = tan(\frac{\theta}{2})$

   So, t is real and thus there is some sequence $\frac{a_n}{k_n} \to t$

   So, $P_n = (\frac{1-(\frac{k_n}{a_n})^2}{1+(\frac{k_n}{a_n})^2}, \frac{2(\frac{k_n}{a_n})}{1+(\frac{k_n}{a_n})^2})$

   $\frac{k_n}{a_n} \to t \Rightarrow P_n = (\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}) = (x, y) = P$