MA 453
Homework b

1. so the gcd of all entries is 1 and $d_1 = 1$

   det$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix} = 4$, det$\begin{pmatrix} -2 & 10 \\ 0 & -4 \end{pmatrix} = 8$, det$\begin{pmatrix} 0 & -4 \\ 3 & -11 \end{pmatrix} = 12$

   so gcd is 4 $\Rightarrow d_1 d_2 = 4$

   det$(M) = (-2)[(-2)(-11) - (-4)(2)] - 0 + 10[(0)(2) - (-2)(3)] = 0$

   so rank = 2

   so $d_1 d_2 = 4$ and rank = 2 $\Rightarrow G = \mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$

2. $\mathbb{Z}/28\mathbb{Z} \times \mathbb{Z}/18\mathbb{Z} \rightarrow$ factorization: $2^2 \cdot 7, 2 \cdot 3^2 \rightarrow \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/9\mathbb{Z} \times \mathbb{Z}/7\mathbb{Z} = \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/9\mathbb{Z} \times \mathbb{Z}/14\mathbb{Z}$

   $\mathbb{Z}/36\mathbb{Z} \times \mathbb{Z}/14\mathbb{Z} \rightarrow$ factorization: $2^2 \cdot 3^2, 2 \cdot 7 \rightarrow \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/9\mathbb{Z} \times \mathbb{Z}/14\mathbb{Z}$

   $\mathbb{Z}/42\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z} \rightarrow$ factorization: $2 \cdot 3 \cdot 7, 2 \cot 3 \rightarrow \mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/42\mathbb{Z} = \mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/9\mathbb{Z} \times \mathbb{Z}/14\mathbb{Z}$

   so all groups are isomorphic

3. Suppose that there is a surjective homomorphism

   define it as $\varphi : \mathbb{Z} \mapsto \mathbb{Z}^2$ and let $x, y \in \mathbb{Z}$ map to (1,0) and (0,1)

   so both x and y are integers so there must be a greatest common divisor g =gcd(x, y) and $x = gx', y = gy'$ with $x', y' \in \mathbb{Z}$

   so both (1,0)=$\varphi(x) = x'\varphi(g)$ and (1,0) = $\varphi(y) = y'\varphi(g)$ are multiples of $\varphi(g)$

   so this means (1,0) and (0,1) lie on the same line generated by $\varphi(g)$ in $\mathbb{Z}^2$, which must be impossible because they are linearly independent

   so no surjective morphism exists.