MA 453
Homework a

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1. (a) For all $i \in I$, we have $e(i) = i$, so $e \in S_n^I$

   (b) Suppose that $\sigma, \sigma' \in S_n^I$

   so, $\sigma(i) = i$ and $\sigma'(i) = i$ for all $i \in I$

   $(\sigma \cdot \sigma')(i) = \sigma(\sigma'(i)) = \sigma(i) = i$

   so, $\sigma \cdot \sigma' \in S_n^I$

   (c) $\sigma^{-1}(i) = \sigma^{-1}(\sigma(i)) = i$

   so $\sigma^{-1} \in S_n^I$

Next, so $S_n^I \rightarrow$ permutations that fix I

so it tells us how it permutes the $n - |I|$ remaining elements.

which is exactly $S_{n-|I|}$

2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 3 & 7 & 1 & 5 & 2 \end{pmatrix}$

In cycle notation: (1 4 7 2 6 5)(3)

For σ^2, is just a shift by 2: (1 7 6)(4 2 5)(3)

For σ^3, is just a shift by 3: (1 2)(4 6)(7 5)(3)

For σ^4, is just a shift by 4: (1 6 7)(4 5 2)(3)

For σ^{-1}, is just a shift by -1: (1 5 6 2 7 4)(3)

3. So the cyclic subgroup of order m can be generated by a cycle of length m.

   so we pick the boring cycle (1 2 3 … 233)

   so, $\langle$(1 2 3 … 233)$\rangle$

4. If $n = \text{ord}(\sigma) \Rightarrow \dfrac{n}{\gcd(n, i)} = \text{ord}(\sigma^i)$

   so, $\dfrac{10}{\gcd(10, i)} = 10$

   so all i such that $\gcd(10, i) = 1$

   so, $i = \{1, 3, 7, 9\}$