MA 453
Homework b Week 1

1.

$+$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\times$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

$+$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

$\times$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

2. 9897654527609805

   $9+8+9+7+6+5+4+5+2+7+6+0+9+8+0+5=90$

   So, it is divisble by 9

   $9+9+6+4+2+6+9+0=45$

   $8+7+5+5+7+0+8+5=45$

   So, the difference is $45-45=0$

   So, it is divisible by 11

   So, the number is divisible by 9 and 11 $\Rightarrow$ divisible by 99

3. Suppose that $a \bmod n = b \bmod n$ and $a' \bmod n = b' \bmod n$, where $a, a', b, b' \in \mathbb{N}$

So, a and b have the same remainder, $a \equiv b \bmod n$

So, $a \equiv b \bmod n \iff n|(a-b) \iff a = b + kn$ for some $k \in \mathbb{Z}$

Similarly, $a' \equiv b' \bmod n \iff n|(a'-b') \iff a' = b' + ln$ for some $l \in \mathbb{Z} \quad (\star)$

Note, $a - b = kn$ and $a' - b' = ln$

So, $aa' - bb' = aa' - ab' + ab' - bb' = a(a' - b') + b'(a - b) = aln + b'kn = n(al + b'k)$

So, $n|aa' - bb$ by the same idea in $(\star) \iff aa' \bmod n = bb' \bmod n$

Next, $(a + a') - (b + b') = (b + kn + b' + ln) - (b + b') = n(k + l)$

So, $n|(a + a') - (b + b')$ by the same idea in $(\star)$

$\iff a + a' \bmod n = b + b' \bmod n$

4. Candidates for G are of the following:

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

$(a, b, c) \to (1, 0, 0) \to ac + b \bmod 2 = 0 \notin G$

$(a, b, c) \to (0, 1, 0) \to ac + b \bmod 2 = 1 \in G$

$(a, b, c) \to (0, 0, 1) \to ac + b \bmod 2 = 0 \notin G$

$(a, b, c) \to (1, 1, 0) \to ac + b \bmod 2 = 1 \in G$

$(a, b, c) \to (0, 1, 1) \to ac + b \bmod 2 = 1 \in G$

$(a, b, c) \to (1, 0, 1) \to ac + b \bmod 2 = 1 \in G$

$(a, b, c) \to (1, 1, 1) \to ac + b \bmod 2 = 0 \notin G$

$(a, b, c) \to (0, 0, 0) \to ac + b \bmod 2 = 0 \notin G$

So, $\{(0, 1, 0), (1, 1, 0), (0, 1, 1), (1, 0, 1)\} \in G$

The only matrix where, $AB = BA$ for all $B \in G$ is the identity matrix $(1, 0, 1)$