1 PROJECT ABSTRACT by Luis Castellanos, Andrej Jakovljevic, Naisha Sarna

Game theory is a branch of mathematics that studies strategic interactions among decision-making players, where the ourcome for each player depends on the choices made by all involved. In competitive situations game theory involve two players whose goal directly oppose each other. In particular we will look into two-player-zero-sum game, one player's gains is exactly the other player's loss. These games can be described by a payoff matrix A, where each row represents a strategy for player 1 and each column represents a strategy for player 2, and the entry A_{ij} tells player 1's payoff when player 1 chooses row i and player 2 chooses column j. Since choosing a single row is often too predictable, players are allowed to choose a mixed strategy, meaning probability distributions over their possible actions. A mixed strategy is represented as a vector x whose entries are nonnegative and sum to 1.

The goal of player 1 is to choose a strategy that performs as well as possible even in the worst case, meaning even if player 2 picks the column that hurts player 1 the most, this leads to the classic minimax problem:

$\min_x \max_j (Ax)_j$

here $(Ax)_j$ is the expected payoff if player 2 chooses column j.

To convert this into an LP problem, we must introduce a variable v which represents the value of the game, in other words it is the payoff player 1 can guarantee no matter what their opponent does. By requiring,

$(Ax)_j \leq v$ for every column j,

we ensure player 1 never receives a payoff worse than v. This then produces the following LP which we can state as:

$\min v$ such that $Ax \leq v, 0 \leq x, 1^T x = 1$

Surprisingly, the corresponding dual LP describes player 2's best mixed strategy. Solving the primal-dual pair gives both player's optimal strategies and the game value, which together form the Nash equilibrium for this zero-sum setting, meaning neither player can improve by changing their strategy.

We will motivate our short formulation with a simple example of Rock-Papaer-Scissors, whose payoff matrix states that Rock drwas with Rock (0), loses to Paper (-1), and beats Scissors (+1), and similarly for other actions. When we convert this game to an LP and solve it, we recover the equilibrium strategy: each player chooses Rock, Paper, and Scissor with probability $\frac{1}{3}$, and the game value is 0, meaning that neither player can obtain an advantage if both play optimally. Our project aims to show how this LP approach can be applied to general minimax problems in game theory.