1.1 LP Formulation

x = tons of bands

y = tons of coils

$$\max \ 25x + 30y$$

$$\frac{x}{200} + \frac{y}{140} \leq 40$$

$$0 \leq x \leq 6000, \quad 0 \leq y \leq 4000$$

<u>Inspection:</u>

Bands: $200\,t/h \times \$25/t = \$5000/h$

coils: $140\,t/h \times \$30/t = \$4200/h$

so, make the 6000 tons of bands and the rest should be coils

$x = 6000 \rightarrow h = 6000/200 = 30h$

$10h$ of coils $\rightarrow$ tons $= 10 \times 140 = 1400$

so, $x = 6000$, $y = 1400$

max $25(6000) + 30(1400) = 192000$

1.2

$x^{c}_{IN} \rightarrow$ Ithaca - Newark, class c

$x^{c}_{NB} \rightarrow$ Newark - Boston, "

$x^{c}_{IB} \rightarrow$ Ithaca - Boston, "

IN: $Y = 300$, $B = 220$, $M = 100$

NB: $Y = 160$, $B = 130$, $M = 80$

IB: $Y = 360$, $B = 280$, $M = 140$

max:

$$300x^{Y}_{IN} + 220x^{B}_{IN} + 100x^{M}_{IN} + 160x^{Y}_{NB} + 130x^{B}_{NB} + 80x^{M}_{NB} + 360x^{Y}_{IB} + 280x^{B}_{IB} + 140x^{M}_{IB}$$

Ithaca $\rightarrow$ Newark

$$x^{Y}_{IN} + x^{B}_{IN} + x^{M}_{IN} + x^{Y}_{IB} + x^{B}_{IB} + x^{M}_{IB} \leq 30$$

Newark $\rightarrow$ Boston

$$x^{Y}_{NB} + x^{B}_{NB} + x^{M}_{NB} + x^{Y}_{IB} + x^{B}_{IB} + x^{M}_{IB} \leq 30$$

$$x^{Y}_{IN} \leq 4, \quad x^{B}_{IN} \leq 8, \quad x^{M}_{IN} \leq 22$$

$$x^{Y}_{NB} \leq 8, \quad x^{B}_{NB} \leq 13, \quad x^{M}_{NB} \leq 20$$

$$x^{Y}_{IB} \leq 3, \quad x^{B}_{IB} \leq 10, \quad x^{M}_{IB} \leq 18$$

Show that for any integer n, $\frac{1}{2n} 2^{2n} \le \binom{2n}{n} \le 2^{2n}$

Let $a_n = \binom{2n}{n}$, $\frac{a_{n+1}}{a_n} = \frac{\binom{2n+2}{n+1}}{\binom{2n}{n}} = \frac{(2n+2)(2n+1)}{(n+1)^2}$

Base case $n=1$: $\frac{1}{2}\cdot 4 = 2 \le a_1 = \binom{2}{1} = 2 \le 4 = 4^1$

Suppose we've checked up to the $n=k$ case then we know that,

$a_k \le 4^k$ (*)

$$a_{k+1} \le \frac{(2k+2)(2k+1)}{(n+1)^2} 4^k$$

$$= 2\,\frac{2k+1}{2k+1}\, 4^k \overset{(*)}{\le} 2 \cdot 2\, 4^k = 4^{k+1}$$

$$\frac{2k+1}{k+1} \le 2 \Rightarrow a_{n+1} \le 4^{k+1}$$

$a_k \ge \frac{4^k}{2k}$ (*)

$$a_{n+1} \ge \frac{(2k+2)(2k+1)}{(k+1)^2} \cdot \frac{4^k}{2k}$$

$$= \frac{(k+1)(2k+1)}{k(k+1)} \cdot 4^k$$

$$= \left(2 + \frac{1}{k}\right)\left(\frac{4n}{2(k+1)}\right) \overset{(*)}{\ge} \frac{4^{k+1}}{2(k+1)}$$

Hence true for the $k = n+1$ case.

2. 1.2

LP formulation:

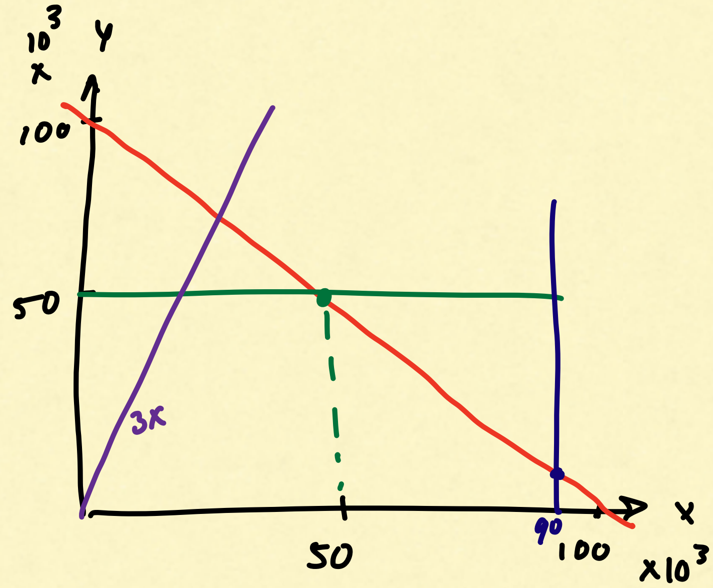
min $0.15x + 0.10y$

$x + y = 100\,000$

$0 \le x \le 90\,000$

$0 \le y \le 50\,000$

$y \le 3x$



$$10^3 \left(\begin{array}{l} 0.15(50) + 0.10(50) = 12.5 \\ 0.15(90) + 0.10(10) = 14.5 \end{array} \right)$$

pick $(50, 50) \times 10^3$

3.

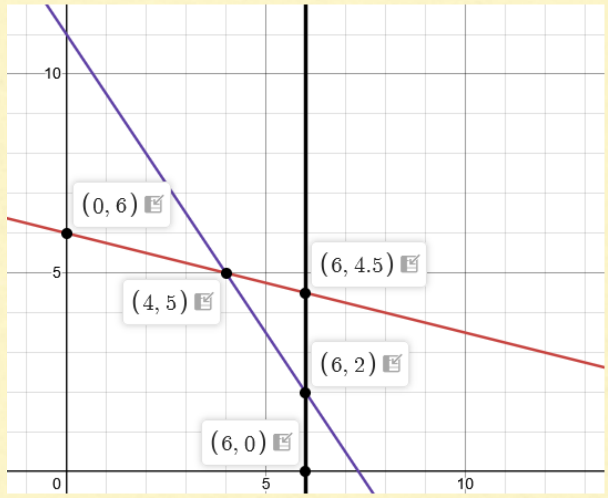
max $5x + 4y$

milk: $3x + 2y \leq 22$

freezer: $x \leq 6$

time: $\frac{1}{4}x + y \leq 6$

$x, y \geq 0$



a) $(0,6)$, $(6,0)$, $(6,2)$, $(4,5)$

$$\begin{array}{cccc} (0,6) & (6,0) & (6,2) & (4,5) \\ \| & \| & \| & \| \\ 24 & 30 & 38 & 40 \end{array}$$

$x, y = 4, 5$

b) Let ice cream price be p

$(0,6)$: $4p + 20 \geq 24 \Rightarrow p \geq 1$

$4p + 20 \geq 6p \Rightarrow p \leq 10$

$4p + 20 \geq 6p + 8 \Rightarrow p \leq 6$

so, $1 \leq p \leq 6$, keeps $(4,5)$ optimal

c) $\xi \rightarrow$ extra gallons

$3x + 2y = 22 + \xi$

$\frac{1}{4}x + y = 6$

$\Rightarrow x = 4 + 0.4\xi$, $y = 5 - 0.1\xi$

$5x + 4y = 40 + 1.6\xi$

Net gain: $40 + 1.6\xi - \xi = 40 + 0.6\xi$

each gallon adds \$.6

until freezer: $x \le 6 \Rightarrow 4 + 0.4\xi \le 6 \Rightarrow \xi \le 5$

so beyond $\xi = 5$ milk ceases to bind

so, yes buy 5 gallons.

profit \$40 $\rightarrow$ \$43